Logic Seminar 2018 Autumn
Homework 3: First-order predicate logic

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1. Express the following sentences in the language of first order predicate logic. If you find that a sentence is ambiguous, try to provide a translation for each interpretation.

Example *John hates everybody who loves him.*

The sentence has (at least) two logically different readings; the pronoun 'him' can either refer to John, or an individual determined by the context of utterance (for instance I point at John's archenemy and utter the sentence.) First we rephrase the sentence according to these readings, so that their logical structure will be revealed:

- (a) For every individual x, if x loves John, then John hates x.
- (b) For every individual x, if x loves y, then John hates x.

(Feel free to skip this step if you are skilled in formalizing.) Now we introduce symbols that stand for the extralogical terms of the sentence:

- a: John
- xRy: x loves y
- xSy: x hates y

Finally we phrase the translations:

- (a) $\forall x\ (xRa \to aSx)$
- (b) $\forall x\ (xRy \to aSx)$

Exercises

1. *Noone loves her.*

2. *If a man loves Mary, he is desperate.*

3. *Some, but not all, of her friends know John.*

4. *Mary hates every woman who loves John.*

5. *Nobody loves someone who hates everyone.*

6. *It is not the case that everyone knows everyone, but everyone knows someone who knows everyone.*

7. *Every man hates a woman who hates every man.*

8. *It is not John that loves Mary.*

9. *Exactly one line can be drawn through any point not on a given line parallel to the given line.*

10. *Three logicians walk in a bar.*

2. Give three examples and three counterexamples for each of the following types of relations either from ordinary language or from the terminology of your favourite area:

serial relation $\forall x\ \exists y\ xRy$

dense relation $\forall x\ \forall y\ (xRy \rightarrow \exists z\ (xRz \wedge zRy))$

Euclidean relation $\forall x\ \forall y\ (\exists z\ (zRx \wedge zRy) \rightarrow xRy)$