Logic Seminar 2018 Autumn

Homework 2

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1 Some of the connectives in standard propositional logic are interdefinable. For instance, "$\varphi \vee \psi$" can be defined with $\wedge$ and $\neg$ as "$\neg(\neg\varphi \wedge \neg\psi)$", as the folowing truth-table shows:

φ	ψ	$\varphi \vee \psi$	$\neg\varphi$	$\neg\psi$	$\neg\varphi \wedge \neg\psi$	$\neg(\neg\varphi \wedge \neg\psi)$
T	T	T	F	F	F	T
T	F	T	F	T	F	T
F	T	T	T	F	F	T
F	F	T	T	T	T	F

Define "$\varphi \wedge \psi$", "$\varphi \vee \psi$", and "$\varphi \leftrightarrow \psi$" with the connectives $\rightarrow$ and $\neg$ in a similar fashion.

2 There are 16 different binary connectives in standard propositional logic, but only 4 of them have symbols. For example, the truth tables of the connectives "none or one" (commonly denoted as NAND) and "because I said so" (commonly denoted as $\top$, and meant so that the sentence is true without respect to the truth values its parts) are as follows:

φ	ψ	φ NAND ψ	$\varphi \top \psi$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	T

Define these two connectives with the standard ones. Find a third one that is expressible in ordinary English, and define that with the standard connectives, too.

4 Consider the following inference:

- $p \rightarrow q$
- $\neg r \rightarrow \neg q$
- $r \rightarrow \neg s$
- $\underline{\neg s \rightarrow t}$
- $\therefore\ \neg t \rightarrow \neg p$

Give an ordinary language argument of this form. Check by means of indirect argument whether or not it is valid.

5 Check the following inference by means of a truth table:

- $(p \vee q) \to \neg r$
- $\underline{p \leftrightarrow \neg q}$
- $\neg r$

Give an ordinary language argument of this form.

6 Check the following logical equivalences by means of truth tables:

(a) $\neg(p \wedge \neg q) \overset{?}{\iff} \neg p \vee q$

(b) $p \rightarrow q \overset{?}{\iff} \neg(\neg p \wedge q)$

(c) $p \vee (q \wedge r) \overset{?}{\iff} (p \vee q) \wedge (p \vee r)$