On Field's Nominalization of Physical Theories

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The Quine-Putnam indispensability argument

(canonized form)

(P1) We ought to have ontological commitment to all and only the entities that are indispensable to our best scientific theories.

(P2) Mathematical entities are indispedsable to our best scientific theories.

(C) We ought to have ontological commitment to mathematical entities.

- (P1) was controversial from the beginning.
- (P2) was considered evident.

Hartry Field was the first who claimed – in his *Science Without Numbers'* (1980) – that premise (P2) is false: mathematical entities are **not indispensable** to our best scientific theories.

Field’s nominalization program

Field claims that a physical theory can be “nominalized”, that is, it can be reconstructed in such a way that

- the new theory does not contain quantifications over mathematical entities
- (still remains attractive)

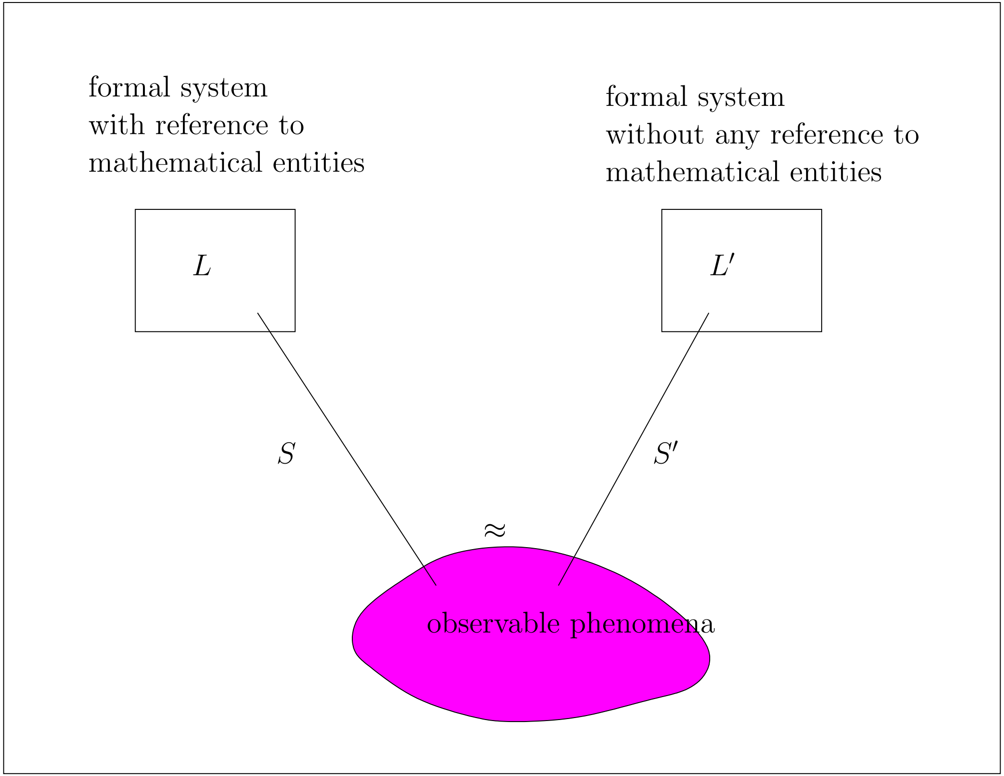
The Nominalization Procedure

1. we have a body of physical facts, in terms of empirical observations

2. we have the usual (platonistic) physical theory describing the observable phenomena in question – containing reference to and quantifications over mathematical entities

3. we construct a new theory which is capable to describe the same phenomena, but without any reference to mathematical entities

Symbolically:



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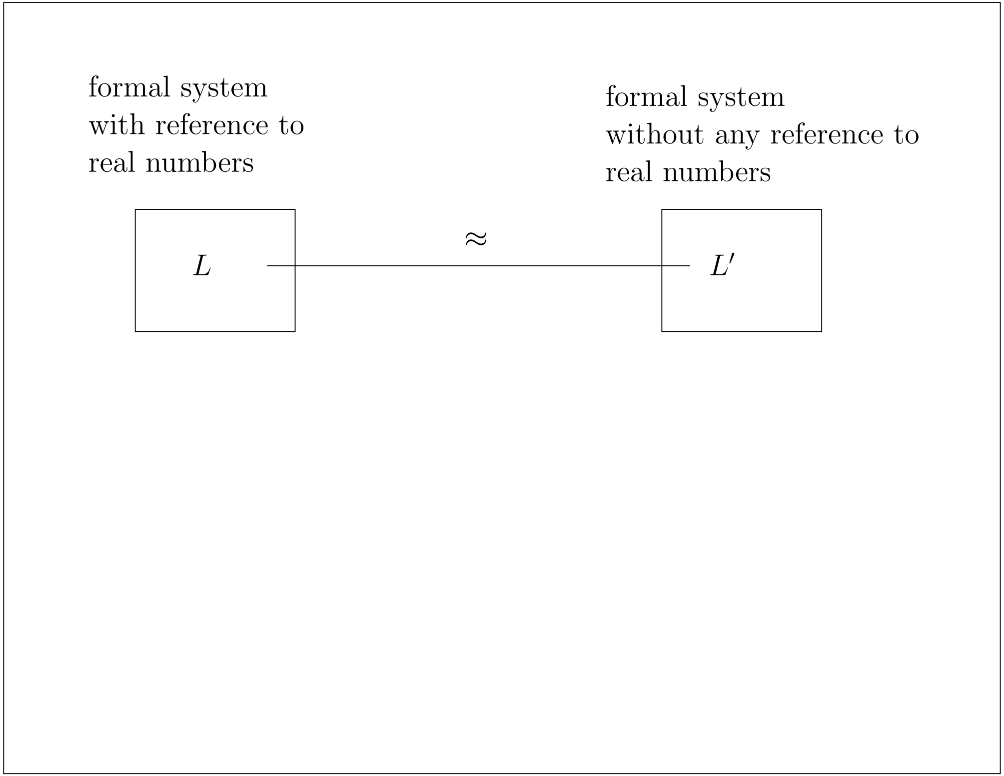


A “toy” physical theory

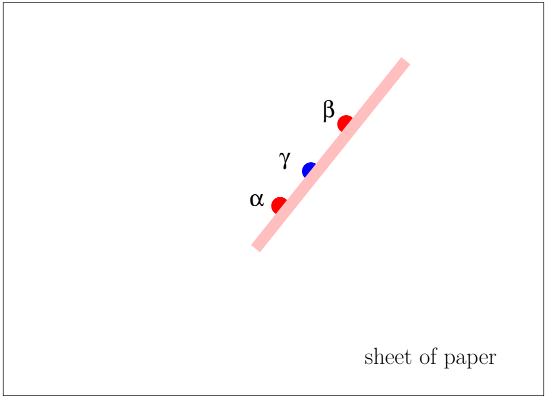
The “toy” physical theory is about a few empirically observable regularities related with the spatial relations of properties of the material points/molecules of a sheet of paper.

Thus:

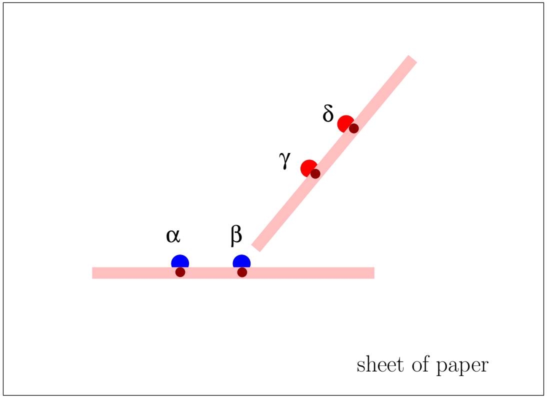
- the only physical entities are the molecules of the paper
- the only measuring equipment is a scale-free ruler

Observed phenomena to be described

Betweenness We say that molecule γ is between molecules α and β if
1) whenever the ruler fits to α and β then it also fits to γ
2) the mark on the ruler corresponding to γ falls between the marks corresponding to α and β.



Congruence We will say that a pair of molecules α, β is congruent to a pair of molecules γ, δ if whenever we mark the ruler at α, β, the same marks will also fit γ, δ.



For example we can observe:

(E1) If molecules α and β are congruent to molecules γ and δ, and γ and δ are congruent to molecules ε and ζ, then α and β are congruent to ε and ζ.

(E2) If we consider three molecules fitting to the ruler, then there is exactly one that lies between the other two.

The usual (platonistic) theory of the paper

The formal system we will use: $L = \left(\mathbb{R}^2, \Gamma, \Lambda\right)$

$\mathbb{R}^2$ pairs of real numbers

$\Gamma(a, b, c)$ relation between three points of $\mathbb{R}^2$ (six real numbers):

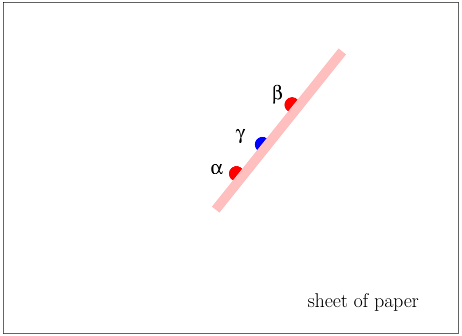
$$\begin{aligned} & \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2} + \sqrt{(c_1 - b_1)^2 + (c_2 - b_2)^2} \\ = \; & \sqrt{(a_1 - c_1)^2 + (a_2 - c_2)^2} \end{aligned}$$

$\Lambda(a, b, c, d)$ relation between four points of $\mathbb{R}^2$ (eight real numbers):

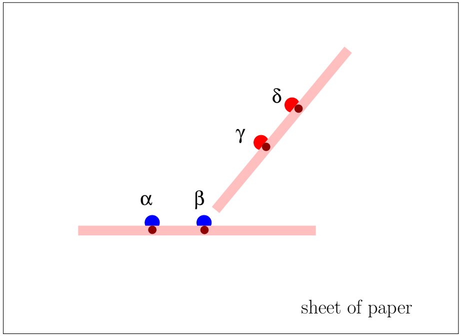
$$\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2} = \sqrt{(c_1 - d_1)^2 + (c_2 - d_2)^2}$$

Semantics

The empirically observable physical configurations, **Betweenness** and **Congruence,** correspond to relations between real numbers, Γ and Λ:



$$\overset{S}{\Longleftrightarrow} \qquad \Gamma(a, c, b)$$



$$\overset{S}{\Longleftrightarrow} \qquad \Lambda(a, b, c, d)$$

Empirical confirmation

The physical theory (L, S), that is the formal system $\left(\mathbb{R}^2, \Gamma, \Lambda\right)$ with the above semantics provides a proper description of our empirical knowledge about the paper. Moreover, it has **predicative** power! For instance, the following is a theorem of $\left(\mathbb{R}^2, \Gamma, \Lambda\right)$:

Theorem 1.

$$\begin{aligned}
&\forall a \forall b \forall g \forall d \forall e \forall z \exists o \; \Gamma(a,d,b) \wedge \Gamma(b,e,g) \wedge \Gamma(g,z,a) \\
&\wedge \Lambda(a,d,d,b) \wedge \Lambda(b,e,e,g) \wedge \Lambda(g,z,z,a) \\
&\longrightarrow \Gamma(a,o,e) \wedge \Gamma(b,o,z) \wedge \Gamma(g,o,d)
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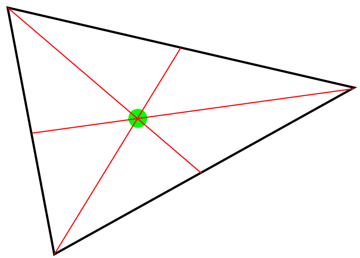

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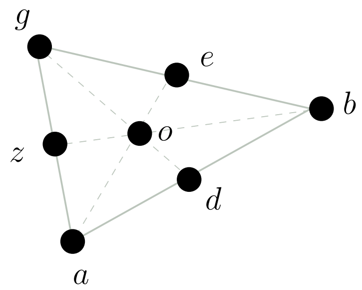

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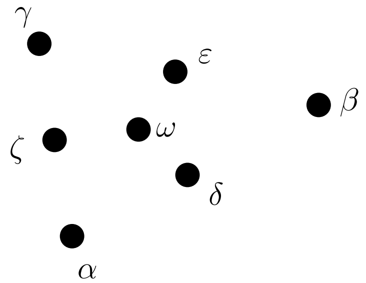
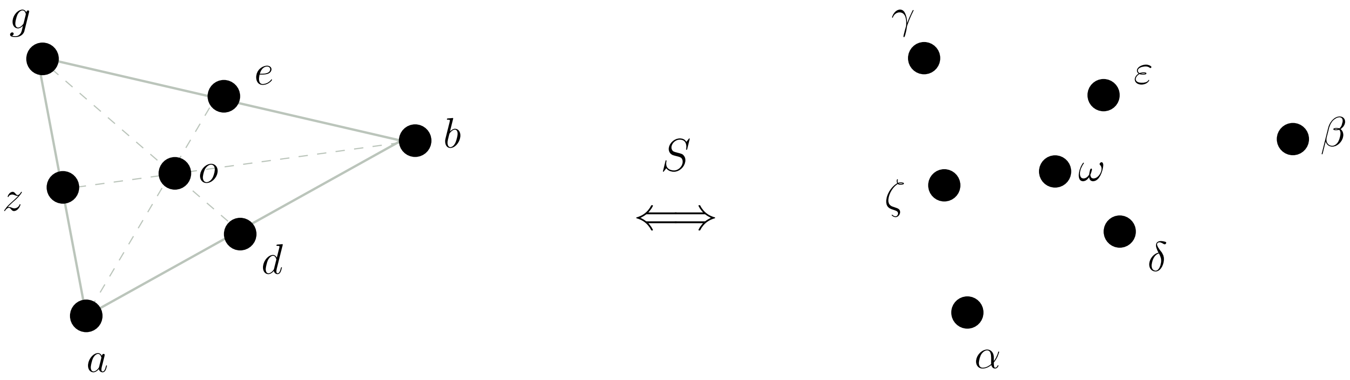
HYPOTHESIS: If molecules $\alpha, \beta, \gamma, \delta, \varepsilon$ and ζ satisfy that δ is between α and β, ε is between β and γ, and ζ is between γ and α, furthermore, α, δ are congruent to δ, β, and β, ε are congruent to ε, γ, and γ, ζ are congruent to ζ, α, then we can always find a molecule ω such that it is between α and ε, and it is between β and ζ and it is between γ and δ.



Indeed!



One can easily verify it on the paper, empirically, by means of the ruler.

(L, S) is, in Field's terminology, a platonistic theory, because it **contains reference to mathematical entities, namely, to real numbers.** (Γ and Λ are relations between real numbers.)

The Nominalized Theory

We will construct another physical theory, that is, we will give another formal system $L^{'}$ with another semantics $S^{'}$, such that:

- it can equally well describe the same observable phenomena
- but without any reference to mathematical entities

The language of $L^{'}$ contains:

- individuum variables: $A, B, C, \ldots$
- a three-argument predicate: Bet
- a four-argument predicate: $Cong$

Beyond the logical axioms of PC(=) (predicate calculus with identity) we give the following "physical" axioms:

T1 $\forall A \forall B \; Cong(A, B, B, A)$

T2 $\forall A \forall B \forall C \; Cong(A, B, C, C) \rightarrow A = B$

T3 $\forall A \forall B \forall C \forall D \forall E \forall F \; Cong(A, B, C, D) \wedge Cong(C, D, E, F)$
$\rightarrow Cong(A, B, E, F)$

T4 $\forall A \forall B \; Bet(A, B, A) \rightarrow A = B$

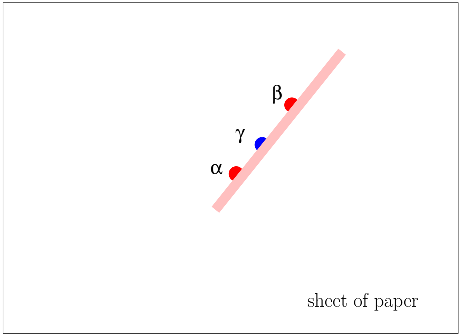
(...)

T10 $\forall A \forall B \forall C \forall D \forall E \forall F \forall G \forall H \; \neg A = B \wedge Bet(A, B, C) \wedge Bet(E, F, G)$
$\wedge Cong(A, B, E, F) \wedge Cong(B, C, F, G) \wedge Cong(A, D, E, H)$
$\wedge Cong(B, D, F, H) \rightarrow Cong(C, D, G, H)$

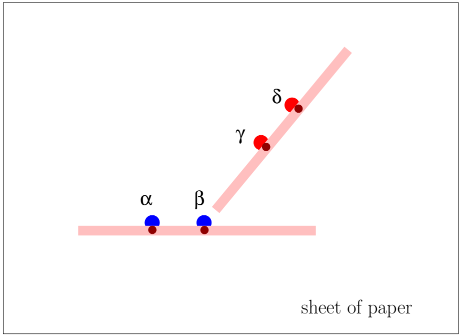
T11 $\forall A \forall B \forall C \forall D \exists E \; Bet(D, A, E) \wedge Cong(A, E, B, C)$

Semantics

Individuum variables $A, B, C, \ldots$ range over the molecules $\alpha, \beta, \gamma, \ldots$ of the paper. The empirically observable physical configurations, **Betweenness** and **Congruence,** correspond to the predicates **Bet** and **Cong**:



$$\overset{S'}{\Longleftrightarrow} \qquad Bet(A, C, B)$$



$$\overset{S'}{\Longleftrightarrow} \qquad Cong(A, B, C, D)$$

Empirical confirmation

The theory so obtained equally well describes our empirical knowledge about the paper. Moreover it has the same **predictive** power! For instance, the following is a theorem of L':

Theorem 1′.

$$\begin{aligned}
&\forall A \forall B \forall G \forall D \forall E \forall Z \exists O \;\; Bet(A, D, B) \\
&\wedge Bet(B, E, G) \wedge Bet(G, Z, A) \wedge Cong(A, D, D, B) \\
&\wedge Cong(B, E, E, G) \wedge Cong(G, Z, Z, A) \\
&\rightarrow Bet(A, O, E) \wedge Bet(B, O, Z) \wedge Bet(G, O, D)
\end{aligned}$$

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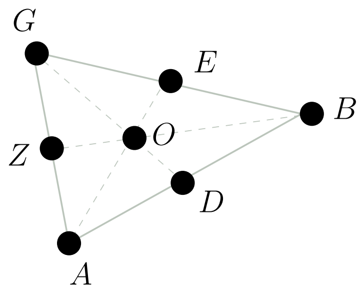

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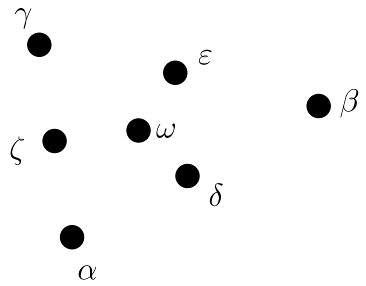

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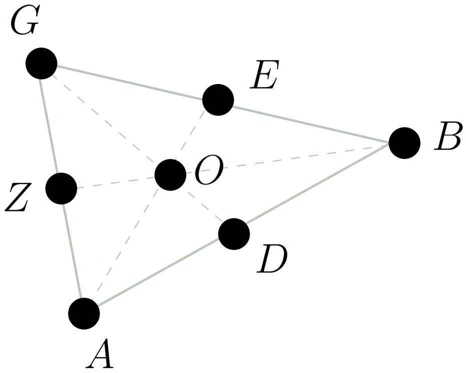
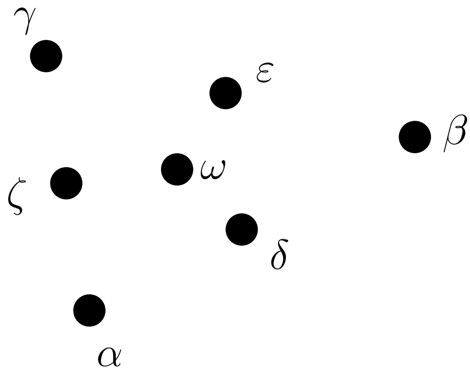


$$\overset{S'}{\Longleftrightarrow}$$



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Concluding remarks

There are however at least two important branches of the philosophy of mathematics which remain unaltered by Field's nominalization results:

- Although we eliminated from the physical theories the references to mathematical entities, we did not eliminated the **mathematical structures** themselves! For example, we eliminated the reference to the points of $\mathbb{R}^2$, but did not eliminate the Euclidean geometry defined by Tarski's axioms T1–T11. And this is enough for the **stucturalist version of platonism**.

- Both the Quine–Putnam argument and Field's nominalization program are based on the tacit assumption that the terms and statements of mathematics have **meanings**, and the only question is the ontological status of the entites that mathematics refers to. This assumption is however unacceptable from the point of view of formalist philosophy of mathematics.