## BTK Filozófia Intézet

## Bírálati lap

Szakdolgozat szerzője:	Brown, William Joseph
Neptun kódja:	W22T02
Születési ideje :	1986.06.07.
Szak:	logika és tudományelmélet
Szakdolgozat címe:	Teljességi eredmények normális modális logikákra Completeness results for normal modal logics
Témavezető :	Gyenis Zalán
Bíráló/beosztása:	Madarász Judit, tudományos főmunkatárs, Rényi Intézet
Érdemjegy:	5, jeles

Szöveges bírálat:

Tanszék:

The subject of the thesis plays a central role both in philosophy and mathematical logic. The thesis is well constructed, coherent, clear and is easy to read.<sup>1</sup> Even though it does not contain new results, it contains detailed proofs of some theorems the full proofs of which seem to be missing from the literature. It introduces most of the notions thoroughly, therefore it is suitable for getting familiar with modal logic. I encourage the author to improve and extend it to a textbook.<sup>2</sup>

The thesis deals with normal multi-modal logics with arbitrarily many modalities of arbitrary ranks.

The first chapter is an introduction to modal logic with really nice examples and explanations.

In the second chapter, it is proved that the weakest normal multi-modal logics are complete w.r.t. Kripke frames. It is also proved that every normal

<sup>&</sup>lt;sup>1</sup>I found incorrect steps in the proofs of Prop.1.7, Lemma 2.13 and in & 2.2, missed some details in the proofs of Lemmas 2.18, 2.22, found some mistakes in the text, statement (1.6) on p.11 was false, found intuitive text on p.51 incoherent. I discussed these problems with the author and he has revised the thesis.

 $<sup>^2 \</sup>rm Pour$ ça, le premier pas pour rait être d'éléver les chapitres 2-ème et 3-ème sur le niveau du premier chapitre.

multi-modal logic is complete w.r.t. its canonical model. In the literature these results are proved for basic modal logics, and the proofs for multimodal logics are thought to be missing. Furthermore, it is proved in full details that there is a normal modal logic with two unary modalities which is not complete w.r.t. any non-empty class of Kripke frames. This proof is not trivial and only its incomplete version can be found in the literature cited by the author.

Is there a normal modal logic with one modality having the same incompleteness property?

The third chapter is on algebraization of modal logic. The Lindenbaum-Tarski algebras of normal multi-modal logics are introduced and it is proved that they are Boolean algebras with operators. The main theorem of the chapter is the Jónsson-Tarski theorem saying that every Boolean algebra with operators can be represented as a set algebra. Thus the Lindenbaum-Tarski algebra of every normal multi-modal logic can be represented as a set algebra. The author, in the Conclusion, claims that this representation provides us a completeness result for any normal multi-modal logic.

Is this completeness result the same as the one in the second chapter saying that every normal multi-modal logic is complete w.r.t. its canonical model? Do we get more than the completeness result in the second chapter?

Are there similar results for non-normal modal logics?

Budapest, 2015. június 1.

Madarász Judit