Richard Dedekind is a German contributor to algebraic number theory.

He is known for Dedekind’s theorem:

Dedekind's theorem[[2]](https://en.wikipedia.org/wiki/Richard_Dedekind#cite_note-2) states that if there existed a [one-to-one correspondence](https://en.wikipedia.org/wiki/One-to-one_correspondence) between two sets, then, the two sets were "similar". He invoked this notion of similarity to give the first precise definition of an [infinite set](https://en.wikipedia.org/wiki/Dedekind-infinite_set): a set is infinite when it is "similar to a proper part of itself," i.e. when it is [equinumerous](https://en.wikipedia.org/wiki/Equinumerous) to one of its [proper subsets](https://en.wikipedia.org/wiki/Subset).

In 1888, Dedekind published a short monograph titled *Was sind und was sollen die Zahlen?* ("What are numbers and what should they be?" Ewald 1996: 790),[[3]](https://en.wikipedia.org/wiki/Richard_Dedekind#cite_note-3) which included his definition of an [infinite set](https://en.wikipedia.org/wiki/Infinite_set).

In 1890, he engaged in a correspondence with Hans Keferstein, a philosophically interested teacher who had published some objections to Dedekind’s work in a paper on the notion of number.

Keferstein’s paper was published in the yearly reports of the Hamburg Mathematical Society.

The letter, which we discuss today was Dedekind’s response to Keferstein’s paper. The correspondence between the two is kept in the State and University Library of Lower Saxony at the University of Göttingen.

I shall draw attention below too some of the passages I consider relevant in the letter

“My dear Doctor,

[…] I should like to ask you to lend your attention to the following train of thought, which constitutes the genesis of my essay […it is] based upon a prior analysis of the sequence of natural numbers just as it presents itself, in experience, so to speak, for our consideration.

What are the mutually independent fundamental properties of the sequence N, that is those properties that are not derivable from one another but from which all others follow?

How should we divest these properties of their specifically arithmetic character so that they are subsumed under more general notions and under activities of the understanding without which no thinking is possible at all but with which a foundation is provided for the reliability and completeness of proofs and for the construction of consistent notions and definitions?

[…]

4) Not every number is a successor n’; in other words phi(N) is a proper part of N. This (together with the preceding) is what makes the number sequence N infinite.

[…]

6) I have shown in my reply, however, that these facts are still far from bein adequate for completely characterizing the nature of the number sequence N. All these facts would hold also for every system S that, besides the number sequence N, contained a system T of arbitrary additional elements t, to which the mapping phi could always be extended while remaining similar and satisfying phi(T) = T. But such a system S is obviously something quite different from our number sequence N, and I could so choose it that scarcely a single theorem of arithmetic would be preserved in it.”

So the question is, what distinguishes the number sequence N and its elements n from a system T of arbitrary additional elements t. **What is number?**

"What then must we add to the facts in order to cleanse our system of such alien intruders t as disturb order in order to restrict it to N?

[…]

This was one of the most difficult points of my analysis and its mastery required lengthy reflection. If one presupposes knowledge of the sequence N of natural numbers and, accordingly, allows himself the use of the language of arithmetic, then of course, he has an easy time of it. He need only say: an element n belongs to the sequence N if and only if starting with the element 1 and counting on and steadfastly, that is, going through a finite number of iterations of the mapping phi, I actually reach the element n at some time; by this procedure, however, I shall never reach an element t outside of the sequence N.

But this way of characterizing the distinction between those elements t that are to be ejected from S and those elements n that alone are to remain is surely quite useless for our purpose; it would, after all contain the most obvious kind of vicious circle.

The mere words ‘finally get there at some time’ of course, will not do either they would be of no more use than, say, the words ‘karam sipo tatura’ which I invent at this instat without giving them any clearly defined meaning.

Thus, how can I, without presupposing any arithmetic knowledge, give an unambiguous conceptual foundation to the distinction between the elements n and the elements t?”

**Questions**

1. What is meant exactly by the words “without presupposing any arithmetic knowledge”?
2. Is this a critique of psychologism?
3. Is Dedekind tacitly acknowledging the need for an essentialist definition of number? (not necessarily Platonism, but essentialism – some *pragmatic* or *semantic* condition for the use and definition of number over and above a purely formal criterion.

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