Monty Hall on the Humean Mosaic

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Abstract

The Monty Hall problem is analyzed without uttering the words "probability", "chance" and "credence". The non-probabilistic description entails that contrary to the received sentiment switching is *not* superior to sticking in the *single-case* Monty Hall game. We consider and reject objections to this conclusion. The most significant objection we shall counter is to the effect that the introduction of probabilistic language can ground the accepted solution of the single-case Monty Hall problem. The philosophical upshot of the present case study is some doubts cast about the tenability of the celebrated *bon mot* according to which "probability is the very guide of life".

Introduction

Many authors in the philosophy literature have come to use the Monty Hall problem to illustrate weighty views on probabilistic reasoning. While everybody agrees that probabilistic arguments do provide justification for the superiority of the switching strategy in the repeated, long-run Monty Hall game, recent debate concerns if there's a way to ground the "normative force of probabilistic reasoning" in a *single case* (Moser and Mulder 1994; Horgan 1995; Baumann 2005, 2008; Levy 2007; Rosenhouse 2009; Sprenger 2010).

The overall message of the present paper will support those (Moser and Mulder 1994; Baumann 2005, 2008) who claim that probabilistic arguments possess no such normative force. However, my argument will reflect a much more thorough skepticism about probability than what these authors were voicing: the reason why probabilistic reasonings fail is that *there's no such a thing as probability*. Whenever probabilistic reasoning is acceptable—e.g. in the *long-run* Monty Hall game—, one can also obtain the accepted result without invoking the notion of probability. The thesis that probability is a notion that is completely eliminable from the scientific discourse was formulated by Szabó (2007, 2010). The present paper can be regarded as an illustration and elaboration of a version of this thesis.

In the first section I provide description of the long-run and single-case Monty Hall games in non-probabilistic terms. As we will see, this description entails that contrary to the received sentiment switching is *not* superior to sticking in the single-case game. In the remaining part of the paper I consider and reject objections to this conclusion. The most significant objection I shall counter is to the effect that the introduction of probabilistic language can ground the accepted solution of the single-case Monty Hall problem.

Ontology of the Monty Hall game

"Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, 'Do you want to pick door No. 2?' Is it to your advantage to switch your choice?" (vos Savant 1990)

Consider n runs of the above Monty Hall game. Assume that the player, throughout the n games, either always switches doors or always sticks to the her original choice. In a given run switching wins against sticking whenever the player happens to pick a door with a goat. Always-switching fares better than always-sticking over the n runs if there are more runs where switching wins. Therefore, always-switching fares better than always-sticking over n runs if

(C) The player initially picks a door with a car less than $n/2$ times.

One way of granting condition C is the following. Assume that

(C_1) The process responsible for the distribution of cars and goats behind the doors is such that over the n runs of the game each door is guaranteed to have the car around $n/3$ times.

(C_2) The distribution of cars and goats behind the doors is, to a good approximation, statistically independent from the distribution of the player's initial door choices over the n runs.

Then, the relative frequency of the player's picking a car is

$$r(\text{car}) \approx r_1 q_1 + r_2 q_2 + r_3 q_3 \approx \frac{1}{3}q_1 + \frac{1}{3}q_2 + \frac{1}{3}q_3 = \frac{1}{3} \tag{1}$$

where r_i is the relative frequency of the ith door having a car, and q_i is the relative frequency of the player's picking the ith door, over the n runs of the game. Since $r(\text{car}) < 1/2$, condition C is satisfied.

C_1 and C_2 are constraints on finite frequencies. What is the warrant for their prevalence? One way of understanding C_2 is as follows. The player's lack of knowledge about the location of the car implies that her initial door choice is not influenced by where the car is. We assume that the host places the car and the goats before the player makes her initial choice, so there's no reverse influence by the player's choice on the location of the car either. Suppose further that besides the absence of direct causal connection there is no relevant common causal type relation—that would again imply a sort of knowledge on the player's part about the car's location, for example by someone telling her before the game where the host will place the car. Hence, the events of the player's picking a door and the host's locating the car are casually separated in each run of the game. Then the Common Cause Principle implies that the distributions of the two events over a long series of runs must be statistically independent (if they would be statistically correlated then they must be causally dependent in some way, but due to the foregoing they are not). And this is what C_2 requires.

C_1 can be a consequence of the host's simple and deterministic algorithm of locating the car and the goats run by run. For example, he may apply the simple rule of placing the car behind door 1 in the first run, behind door 2 in the second run, door 3 in the third, then door 1 again, and so on, granting the 1/3 relative frequency for each door. However, C_1 may also obtain as a result of a "random" process responsible for the car's and goat's locations. To see how this happens, suppose the host throws a die in every run. If it comes up 1 or 2, he puts the car behind door 1, if it comes up 3 or 4, behind door 2, if it's 5 or 6, behind door 3. Imagine the space of initial conditions pertaining to the dynamics of throwing the die. Assume that the die is fair in the sense that

(F_1) The phase space volumes of those initial conditions that lead to a particular outcome are equal, namely 1/6 of the total phase volume of the space of available initial conditions.

(F_2) Any initial condition has a sufficiently small neighborhood such that this neighborhood can be divided into 6 cells of equal volume, each of which leading to one of the 6 outcomes. (Cf. Kapitaniak et al. 2012)

The overall implication of F_1 and F_2 is the following. Conceive the space of initial conditions of throwing the die as a statistical ensemble. Each member of the ensemble has the property of being '1', '2', '3', '4', '5' or '6', corresponding to the outcome that the initial condition in question leads to. The host's throwing the die *n* times over the *n* runs of the game picks an *n* element sample of this ensemble. Due to F_2 the die throw is uncontrollable for the host: he won't be able to know which outcome will be the result of the initial condition generated by his throw. What follows from this "lack of knowledge" is that the outcome properties, '1', '2', ..., '6', of the initial conditions will not have influence on which initial condition will be realized by the host's throw—quite analogously to the player's situation where the location of the car doesn't have influence on the player's door choice due to her "lack of knowledge" about the car's location. In other words, assuming the absence of direct as well as common cause type relation, the two sorts of properties of the points of the initial condition space—the outcome properties and the property of having or not having been selected in the *n* element sample—are causally independent for each point. Then, by the Common Cause Principle, these properties must be statistically independent over the ensemble of initial conditions. Statistical independence means that the relative ratio of initial conditions leading to a particular outcome among the points that have been selected in the sample will be about the same as in the original ensemble of initial conditions, namely, by assumption F_1, around 1/6. That is, the relative frequency of each outcome in the series of *n* die throws is going to be around 1/6. This implies that each door will have the car around $n/3$ times over the *n* games, which is what C_1 requires.

C, C_1, C_2, F_1 and F_2, as well as the causal independence of the events in question together with the validity of the Common Cause Principle, are objective conditions under which always-switching is better than always-sticking in the long-run Monty Hall game. While in the above analysis the notion of absence of casual influence has been grounded in "lack of knowledge", it must be emphasized that the mere lack of knowledge on the player's part about the location of the car does not itself guarantee the objective conditions under with always-switching is better. The player's lack of knowledge about the car's location doesn't imply that the distribution of cars behind the doors is uniform, conforming to C_1. On the contrary, in order to know that always-switching indeed fares better, the player must have evidence about the actual distribution, for example in the form of information about the host's method of locating the car by throwing a fair die, characterized by F_1 and F_2. More generally:

Ontic principle of rational choice *It is rational for the player to favor one strategy over the other if and only if she has evidence about the* *objective conditions under which one strategy is better than the other.*

In line with this principle, knowing the rules of the game, together with the host's algorithm of locating the car by means of a fair die, plus having background knowledge about the relevant laws of nature, including the dynamics of die throwing and the Common Cause Principle, it is rational for the player to favor always-switching over always-sticking in the long-run Monty Hall game.[1]

Condition C, the requirement under which always-switching wins in the long run, was based on the condition under which switching wins in each individual run. The condition is this:

(c) The player initially picks a door with a goat.

Notice that condition C in general doesn't imply that condition c is satisfied in any given individual run; nor do its subconditions C_1, C_2. That is, the condition under which switching is better than sticking in a single run will not be guaranteed by the conditions under which always-switching is better than always-sticking in the long run. Combining this simple fact with the application of the Ontic principle of rational choice for the single-run game—the principle that provided the accepted solution in the long-run Monty Hall game—, one receives:

(R) If the player has only evidence of condition C, but not of condition c, it is not rational for her to favor switching over sticking in a single run.

This conclusion obviously contradicts to the received sentiment on what's rational to do in the Monty Hall game. In what follows I consider three objections to the effect that result R must be seen as incorrect. I shall argue that none of these objections is cogent.

How long is the long run?

Conclusion R means that in general there is a divergence between what's rational to do in a single case and what's rational to do in the long run. To some authors the mere fact of this appears suspicious. They ask (Rosenhouse 2009, p. 164; Sprenger 2010, p. 337): given that the long-run game is just a collection of single run games, how can it possibly be that what's rational in the single case doesn't match what's rational in the long run? For how large n one should start to see a long run and favor switching? How long is the long run? They take it that these questions have no unambiguous answer.

In fact, precisely the observation that a long-run game is nothing but a collection of single-run games is what grounds the answer to these questions. For this also means that the single-run game can be accounted for as a special case of the long-run game when $n = 1$. Indeed, condition c, under which switching wins in a single run, is just a special case of condition C, under which always-switching wins in the long run, when $n = 1$. However, as we shall now make clear, conditions C_1 and C_2 will only grant the satisfaction of C when n is "large", but not when n is "small" (in particular, not when $n = 1$)—which is what explains why it's rational, knowing that C_1 and C_2 obtain, to favor always-switching in the long run, but not to favor switching in a single run.

The formulation of C_1 and C_2 has an ambiguity: it contains the vague notions of "*around* $n/3$ times" and "*to a good approximation* statistically independent". Let us now make this more precise:

[1]Of course, knowing any other set of conditions that grants C, for example C_1–C_2 as mere facts about finite frequencies, is sufficient to make it rational to favor always-switching in the long-run Monty Hall game.

(C_1') The process responsible for the distribution of cars and goats behind the doors is such that over the n runs of the game each door is guaranteed to have the car with relative frequency $1/3$, with $O(1/n)$ precision.

(C_2') The distribution of cars and goats behind the doors is statistically independent from the distribution of the player's initial door choices over the n runs, with $O(1/n)$ precision.

C_1' and C_2' express that the values of the relative frequencies in question can be approximated as follows:

$$\frac{1}{3} - \frac{K_1}{n} \leq r_i \leq \frac{1}{3} + \frac{K_1}{n} \tag{2}$$

$$-\frac{K_2}{n} \leq \rho_{ij} \leq \frac{K_2}{n} \tag{3}$$

with correlation coefficient

$$\rho_{ij} = \frac{s_{ij} - r_i q_j}{\sqrt{r_i(1-r_i)}\sqrt{q_j(1-q_j)}} \tag{4}$$

where r_i is the relative frequency of the ith door having a car, q_j is the relative frequency of the player's picking the jth door, and s_{ij} is the relative frequency of the conjunction of the former two events, over the n runs of the game. K_1 and K_2 are positive constants not depending on n.

(2)–(3) implies that the relative frequency of the player's picking a car is 1/3 with $O(1/n)$ precision, that is

$$\frac{1}{3} - \frac{K}{n} \leq r(\text{car}) \leq \frac{1}{3} + \frac{K}{n} \tag{5}$$

where K is a positive constant expressible in terms of K_1 and K_2. (5) is just a more precise formulation of the approximate equality (1). According to condition C, always-switching wins if $r(\text{car}) < 1/2$, that is, using the estimate for $r(\text{car})$ given by (5),

$$\frac{1}{3} + \frac{K}{n} < 1/2 \tag{6}$$

which implies

$$n > 6K \tag{7}$$

Hence, C_1' and C_2' only grant C, the condition of always-switching being better then always-sticking, when the number of runs is larger than $6K$. Knowing that C_1' and C_2' obtain, in line with the Ontic principle of rational choice, it is thus only rational for the player to favor always-switching if the series of repeated games is longer the $6K$ runs. This threshold is what can be seen as the definition of "long run".

As we saw in the previous section, the validity of statistical conditions C_1 and C_2 derive from the Common Cause Principle (in the absence of certain causal dependencies). In fact, it is the validity of conditions C_1' and C_2', together with the numeric values of K_1, K_2, and hence that of K, that are dependent on the Common Cause Principle. The magnitude of constants K_1, K_2 and K must be seen as something determined by the "strength" of statistical independence that follows from the satisfaction of the Common Cause Principle, more precisely, by how rapidly frequencies converge to the value pertaining to statistical independence as a consequence of the principle's prevalence. This is a matter of contingent fact, characterizing the statement of the Common

runs	door with the car	player's door choice	$r(\text{car})$
1	1	1	1
2	2	1	0.5
3	3	1	0.33
4	1	1	0.5
5	2	1	0.4
6	3	1	0.33
7	1	1	0.43
8	2	1	0.38
9	3	1	0.33
10	1	1	0.4
11	2	1	0.36
12	3	1	0.33
13	1	1	0.38
14	2	1	0.36
15	3	1	0.33

Table 1: *15 runs of a repeated Monty Hall game. The host applies the "door 1, door 2, door 3, door 1, door 2, door 3,..." rule for locating the car, and the player picks door 1 in each run. The relative frequency of the player's picking a car converges to 1/3, but only after four runs does it get below 1/2, which is the condition of always-switching being better than always-sticking*

Cause Principle, to be investigated by empirical means. For example—reversing the argument for C_2 in the previous section—, the precision with which frequencies conform to value 1/6 in cases of repeatedly throwing a fair die, is a fact to be regarded as a characterization of the Common Cause Principle's statement about those physical situations.

Nevertheless, one can get a sense of the magnitude of threshold given by (7) on a simple example. Suppose that the host decides to apply the "door 1, door 2, door 3, door 1, door 2, door 3,..." rule for locating the car run by run, and the player happens to pick door 1 in each run. Clearly, C'_1 and C'_2 are satisfied in this case, and the relative frequency of the player's picking a car will conform to 1/3, granting condition C. However, as is depicted in Table 1, only after four runs does condition C come to hold. More precisely, it is only after four runs that the value of $r(\text{car})$ gets below 1/2 and stays there stably. Therefore, always-switching is only guaranteed to win after $n = 4$ runs.

Of course, one may not assume that an agent playing the game has precise knowledge about the values of K_1, K_2, K, and hence about the threshold given by (7). However, the player may well have knowledge about typical values of these parameters, based on everyday experience. For example, she may well expect that throwing a fair die a thousand times will never result in less then a hundred of outcome '6'. Based on knowledge of this sort of objective conditions, in perfect harmony with the Ontic principle of rational choice, it is rational for the player to favor always-switching if the number of runs is, say, 1000, but it is not rational to do so when the number of runs is 1, that is in the single-case game.

Many doors

Conclusion R is in harmony with the first intuition of most people according to which it is indifferent whether to switch or stick in the single-case Monty Hall game. However, there is a well-known variant of the standard Monty Hall problem where intuition unequivocally favors switching in the single case. Suppose that instead of 3 there are many, say 1000 doors, one of them having the car and the remaining 999 having goats. The player picks a door, then the host opens all but one of the 999 doors left, each of them with a goat. Now the player can choose: whether sticking with her original door or switching to the one left unopened. Most people realize that it is "very unlikely" to pick from 1000 doors the one with the car, so it's worth switching to the door left after the host reveals 998 goats. It is taken for granted that this insight also shows why switching is to be favored *in the original 3 door game* (vos Savant 1990; Rosenhouse 2009, p. 37).

Let me reconstruct what I think is behind the many doors intuition. Conceive the 1000 doors as a statistical ensemble. In a single run of the Monty Hall game each member of this ensemble acquires two properties: having a car or a goat, and having or not having been picked by the player. Due to the player's lack of knowledge about the car's and goats' locations, these two properties are causally independent for each door. Then, by the Common Cause Principle, these properties must be statistically independent over the ensemble of the 1000 doors. In other words, if the player chose the door with the car, that would mean a correlation—a correlation of her door choice and the location of the car—without a causal explanation. If the Common Cause Principle is true in our world, such an accident is not possible, hence the player picks a goat, and the door left unopened by the host must hide the car, which is why the player should switch.

Notice however that this reasoning is only valid in case of many doors but not in case of a few ones. This is because picking the door with the car will only imply statistically significant correlation when the statistical ensemble of doors is sufficiently large. Denote by N the number of doors, and by C and P the properties 'having a car' and 'having been picked by the player', that the doors may posses. Let $R(C)$, $R(P)$ and $R(C\&P)$ be the relative ratios of these properties over the ensemble of the N doors. Now, in complete analogy with condition (3) formulated in terms relative frequencies over the ensemble of runs in a long-run game, statistical independence of properties C and P means that their correlation coefficient, defined in terms of relative frequencies calculated over the ensemble of doors in a single-run game, satisfies:

$$-\frac{\kappa}{N} \leq \rho\,(C,P) \leq \frac{\kappa}{N} \tag{8}$$

where κ is a positive constant, and correlation coefficient $\rho\,(C,P)$ is given by

$$\rho\,(C,P) = \frac{R(C\&P) - R(C)\,R(P)}{\sqrt{R(C)\,(1-R(C))}\sqrt{R(P)\,(1-R(P))}} \tag{9}$$

$$= \begin{cases} \dfrac{\frac{1}{N}-\frac{1}{N^2}}{\frac{1}{N}\left(1-\frac{1}{N}\right)} = 1 & \text{if the car is picked} \\[2ex] \dfrac{0-\frac{1}{N^2}}{\frac{1}{N}\left(1-\frac{1}{N}\right)} = -\dfrac{1}{N-1} & \text{if a goat is picked} \end{cases} \tag{10}$$

No matter what the value of κ is, it is clear that if N is sufficiently large then $\rho(C,P) = 1$ is going to violate condition (8), which means that the player's picking the car will indeed correspond to there being correlation between C and P. At the same, $\rho(C,P) = -1/(N-1)$ implies that for large N $\rho(C,P)$ scales with $1/N$, which is just to say that it satisfies (8), and so the player's picking a goat is compatible with C and P being statistically independent. On the other hand, if N is small $\rho(C,P)$ will have about the same order of magnitude regardless of whether a goat or the car is chosen by the player. In particular, in case of $N = 3$, $\rho(C,P) = -1/(N-1) = -1/2$ if the player picks a goat, which is of the same order of magnitude as 1, the value of $\rho(C,P)$ when the player picks the car, both values being compatible with (8). That is to say, for 3 doors, in harmony with our intuition, the player's choosing the door with the car doesn't necessitate an unexplainable correlation, because it doesn't necessitate correlation in the first place, and so the argument for switching applicable for a single run, based on the statistical ensemble of doors and the Common Cause Principle over it, doesn't go through.

Probability

The notion of probability played no role whatsoever in what has been said so far. The above considerations were based on reference to

- relative frequencies over finitely many runs
- conditions about these frequencies taking certain values with finite, $O(1/n)$, precision, like C'_1 and C'_2
- relative ratios, such as quantities like $\frac{\#\text{cars}}{\#\text{doors}}$, $\frac{\#\text{goats}}{\#\text{doors}}$, or ratios of phase space volumes
- conditions expressible in terms of these ratios, like that of F_1 and F_2
- the notion of causal independence
- the Common Cause Principle, as a statement relating regularities and casual structure—the former being understood as finite frequencies taking certain values with finite precision
- the notion of an agent having/lacking knowledge about conditions of the above kind

None of these ingredients refers to the notion of probability.

Many will contend that this is exactly the reason why conclusion R is not satisfactory. Recall that the derivation of R was based on the application of the Ontic principle of rational choice. One might object that the 'only if' part of this principle is just not correct: one may very well have rational grounds to favor a strategy over another even though one doesn't have evidence about the conditions under which one strategy is *determined* to be better than the other. For one may still have evidence about one strategy being *more likely* to be better than the other. More precisely, consider the following two probabilistic principles of rational choice:

> *It is rational for the player to favor one strategy over the other if she has evidence about the conditions under which one strategy has more objective chance to win than the other.*

It is rational for the player to favor one strategy over the other if she has higher credence in the conditions under which one strategy wins than in the conditions under which the other one wins.

Using these principles, one might argue as follows (Sprenger 2010, pp. 338–339). Assume the host throws a fair die to decide on the car's location. If it comes up 1 or 2, he puts the car behind door 1, if it comes up 3 or 4, behind door 2, if it's 5 or 6, behind door 3. Since each outcome of throwing a fair die has chance 1/6, each door is going to have 1/3 chance of hiding the car. Now apply the standard probabilistic reasoning to obtain that the chance of winning the car by switching is 2/3, while it is 1/3 by sticking. Assuming that the player has knowledge about these chances, according to the first of the above principles of rational choice, it is rational for the player to favor switching. Alternatively, use the Principal Principle at any point of the argument to see that the player should have credence 2/3 in winning by switching, and 1/3 in winning by sticking. According to the second of the above principles, again, it is rational to favor switching in the single-case Monty Hall game—in contradiction with conclusion R.

My argument against this line of reasoning will be based on a sort of (in)dispensability principle à la Quine–Putnam: we must not have ontological commitment to things dispensable to our best scientific theory. This ontological tenet will be coupled with an (in)dispensability principle regarding norms: whatever norms we have about rational behavior, these norms must be expressible in terms of those things that we are ontologically committed to, hence, without referring to things dispensable to our best scientific theory.

Consider a die throw. Assume that each outcome has objective chance 1/6. The standard probabilistic derivation of the always-switching solution in the *long-run* Monty Hall game makes use of there being a correspondence between this single-case chance and long-run frequency: in a long sequence of repeatedly throwing the die each outcome will occur around 1/6 of the times. This fact is usually justified by referring to the law of large numbers, according to which the chance that long-run frequency is close to the value of the corresponding single-case chance is nearly 1, plus to Cournot's principle according to which an event with chance nearly 1 is certain to happen (see e.g. Galvan 2008, sec. 3). What we thus have here is an *explanation* of long-run frequency in terms of single-case chance (cf. Emery 2017).

Notice however that we have already seen an explanation of why frequencies conform to 1/6 when repeatedly throwing a fair die. In the second section this fact was derived from F_1 and F_2, and the Common Cause Principle. This derivation doesn't refer to chance. So we are facing two different, competing explanations, invoking different vocabularies, for one and the same thing. What we observe here is similar to the problem of mental causation in the philosophy of mind: the behavior of a conscious agent seems to be explainable in two different ways, in terms of different vocabularies, one invoking mental states and one invoking physical states. In accordance with the dispensability principle, many take this fact as evidence of mental states being eliminable from our ontological picture—mental states being, in some sense, identical with, or reducible to, physical states. Quite analogously, facts about chance over-explain the (non-probabilistic) behavior of the die, which can be fully understood in terms of its non-probabilistic description. In line with the dispensability principle, this suggests that we should eliminate the notion of chance from the narrative we give about the behavior of the die. Together with the normative version of the dispensability principle this also means that whatever reasoning we provide about what one ought to do, or what's rational to do, in cases involving die throws, this reasoning should be free—should be able to be made free—of any reference to chance. I think this is the essence of why the above probabilistic argument in favor of the switching solution in the *single-case* Monty Hall game cannot be correct—while the superiority of switching in the long run can be established without relying on "chance facts", as we saw it in the previous sections.

What we said about the case of throwing a die applies rather generally. To see this, consider another example. Suppose an urn contains 1000 balls. 500 of them are black, 500 are white. Someone draws a ball blindfolded. What is the chance of picking a white ball? The standard answer is 1/2. This answer is taken to explain why the relative frequency of picking a white ball over a long series of repeated draws is expected to be around 1/2. However, we also have an explanation of this fact that doesn't refer to chance. This can be grounded in the following:

(U_1) Half of the balls in the urn are white.

(U_2) The drawing being blindfolded implies that the fact as to which ball is picked is causally uninfluenced by the color of the balls (assume the absence of direct and common casual relations).

(U_3) Consider the ensemble of balls in the urn. Some of them are white, some of them are black. Suppose that a long series of repeated draws is performed. By this, each ball in the box will acquire the further property of having been picked or not having been picked over the series of draws. Due to U_2, this property of the balls is causally independent from their colors. Then the Common Cause Principle implies that these two kinds of property—whether having been picked and whether being white—must be statistically independent over the ensemble of balls in the urn. Statistical independence means that the relative ratio of white balls among the ones that have been picked must be about the same as in the original ensemble of balls in the urn, namely, due to U_1, 1/2. That is, the relative frequency of white balls in the series of draws is going to be around 1/2.

What's common to the urn and die cases is the following. We are given a statistical ensemble, over which certain properties are distributed—the ensemble of balls in the urn with color properties 'black' and 'white'; the ensemble of initial conditions of throwing a die with outcome properties '1', '2', '3', '4', '5' and '6'. We pick a large sample from this ensemble. If the process of sampling is causally autonomous from the relevant properties of the members of the ensemble, then, due to the Common Cause Principle, the relative ratios of the various properties over the original ensemble will be reflected by the relative frequencies of properties in the sample. This is the explanation of why stable frequencies emerge. Such an explanation is available whenever the frequencies in question can be accounted for as results of sampling an ensemble with pre-established properties.[2] Such an explanation dispenses with any reference to the notion of chance.

In U_1–U_3 we described the ontology of a random trial like picking balls from an urn. If there's any reason to use the notion of chance in characterizing such a trial

[2] The question arises as to whether frequencies emerging from quantum phenomena can be given such an explanation. Notice that the sampling account provides a kind of deterministic hidden variable theory of the frequencies in question—where the hidden variables are just the properties over the ensemble being sampled. While standard no-go results exclude the possibility of hidden variable theories of a *desired sort*, it is known that quantum mechanical frequencies *can* be accounted for by a deterministic hidden variable theory (that is nonlocal and contextual). Such a theory is Bohmian mechanics.

(saying, e.g., "the chance of picking a white ball is 1/2"), *it must be expressible in the non-probabilistic terms of U_1–U_3*. In fact, this is also true for the notion that the trial in question is "random". Compare the urn case with a slightly modified scenario. The one who draws the ball is not blindfolded but can check the ball's color before picking one. She likes white balls more. It is clear that the setup doesn't qualify as "random" any longer. What renders the trial "random" is, I believe, nothing but the satisfaction of condition U_2, that is the process of sampling being causally independent from the properties to be sampled. Indeed, this is the condition under which the player's initial door choice in the Monty Hall game can be regarded as a random choice—her picking a door not being influenced by the doors' property 'having a car'. And this is what renders throwing a die a random experiment—the unpredictability of the dynamics characterized by F_2 implies that the initial condition realized by the agent's throw is not influenced by the outcome properties of the initial conditions. Notice that this notion of randomness doesn't refer to the prior concept of chance.

If the sampling is random in the above sense, the emerging relative frequencies are guaranteed to be equal with the relative ratios of the corresponding properties over the ensemble being sampled, via the mechanism in U_3. The relative frequency of picking a white ball will be equal with the relative ratio of white balls in the urn. The relative frequency of the die coming up, say, 6 will be equal with the relative ratio of initial conditions leading to outcome '6' in phase space. This value—1/2 in the urn case and 1/6 in the die case—is what can be defined as the value of chance in the respective random trial. Note that while frequency is related to a series of repeated trials, the relative ratio of properties over the ensemble in question is a single-case notion in the sense that this ratio characterizes each individual run of the trial. This provides a sense in which chance can be regarded as single-case. It must be emphasized that the equality of the value of chance and the corresponding single-case ratio is not a matter of contingent fact, expressing a contingent equality of two independent quantities, but rather a consequence of the way we *define* what "chance" means in the context of the random trial at hand.[3]

The above definition of chance makes clear why it's unreasonable to link rational behavior in a single case to the value of chance, while such a link is completely justified when it comes to rational choice in the long run. Assume that in the Monty Hall game "the mechanism which assigns the prize to a specific door is not biased in favor of a particular door" (Sprenger 2010, p. 338). That is, assume that the mechanism which assigns the prize to a specific door is a random process, and the chance of each door being assigned the prize is equal—"random" and "chance" being understood as defined above. Since the value of chance equals with the value of the emerging long-run frequency in the repeated game, each door having equal chance of hiding the car implies C_1, the condition that the distribution of cars behind the doors is uniform, under which always-switching wins in the long run. Thus, the appropriate value of chance indeed expresses such a condition the knowledge of which, in line with the Ontic principle of rational choice, makes it rational for the player the favor one strategy over the other in the *long-run* Monty Hall game. At the same time, the value of chance, as a single-case quantity, equals with the relative ratio of the corresponding property over the ensemble in question. For example, if the car's location is decided by throwing a die, the chance that a certain door will receive the car equals with the ratio of those initial conditions of throwing the die which, if realized, will get the car behind the door in question. This ratio of phase space volumes is a well-defined physical quantity in

[3] The definitional character of our chance assignments is pointed out in Szabó 2007.

each individual run of the game, but it tells us nothing about the fact as to which initial condition is realized, and thereby which door receives the car, *in a given run*. Hence, chance as a single-case notion tells us nothing about the condition, c, under which switching wins in a single run. One might wish to say that even though the phase volume ratio in question tells nothing about whether switching is *determined* to win or not, it does tell us about the *chance* of switching being better than sticking in a single run. Notice however that since single-case chance is identified with the corresponding phase volume ratio *by definition,* this would amount to be a simple tautology saying that the phase volume ratio in question equals with the phase volume ratio in question. If a notion, chance, is eliminable from our discourse, then the introduction of this term, "chance", by means of a definition cannot possibly allow one to formulate a condition, whether expressing a fact or a norm about what's rational to do, that is not formulable without the usage of this notion. This is essentially what our dispensability principles claim. Note that this observation remains true even if the notion of chance, a shorthand for a non-probabilistic expression, is further identified, through the Principal Principle, with "credence".

What we can observe here is a kind of Humean supervenience. Probabilistic facts supervene on non-probabilistic facts, on the pattern of the "Humean Mosaic". This should also mean that norms regarding what's rational to do, to the extent to which norms are governed by facts, must also supervene on the Humean Mosaic. This thesis might be coined as the "Humean supervenience of the very guide of life".

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