A focused framework for emulating modal proof systems

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Outline

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A focused framework for emulating modal proof systems

1. Focusing
2. The general framework
3. Emulation of modal proof systems

Some background: Focused proof systems

Let's consider a (1-sided) sequent system setting.

- Better organize the **structure** of derivations.
- Emphasis on: non-invertible vs. invertible rules.
- Propositional connectives have:
  - a positive version;
  - a negative version.

$$\frac{\vdash \Theta, B_i}{\vdash \Theta, B_1 \vee B_2}\ \vee^{+} \qquad \frac{\vdash \Theta, B_1, B_2}{\vdash \Theta, B_1 \vee B_2}\ \vee^{-}$$

Some background: Focused proof systems

Let's consider a (1-sided) sequent system setting.

- Better organize the **structure** of derivations.
- Emphasis on: non-invertible vs. invertible rules.
- Propositional connectives have:
  - a positive version;
  - a negative version.
- Polarization of a formula does not affect its **provability**.

Some background: Focused proof systems

store

$\vdash \Theta \Uparrow \Gamma$

release

$\vdash \Theta \Downarrow A$

decide

Some background: Focused proof systems

store (a positive formula to possibly focus on later)

$\vdash \Theta \Uparrow \Gamma$ $t^-, f^-, \vee^-, \wedge^-, \forall$

release

$\vdash \Theta \Downarrow A$ $t^+, f^+, \vee^+, \wedge^+, \exists$

decide (on a positive formula to focus on)

Some background: Focused proof systems

store (a positive formula to possibly focus on later)

$\vdash \Theta \Uparrow \Gamma$ **NEGATIVE PHASE (invertible)**

release (change of phase)

$\vdash \Theta \Downarrow A$ **POSITIVE PHASE (non-invertible)**

decide (on a positive formula to focus on)

Some background: Focused proof systems

store

$\vdash \Theta \Uparrow \Gamma$

release

$\vdash \Theta \Downarrow A$

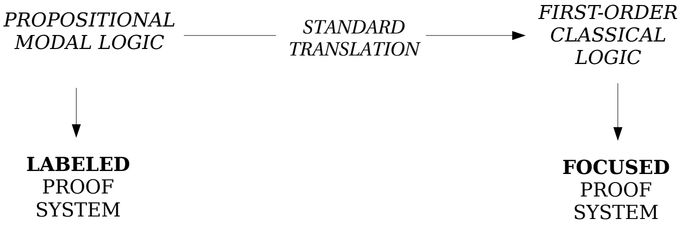
decide

By the way,

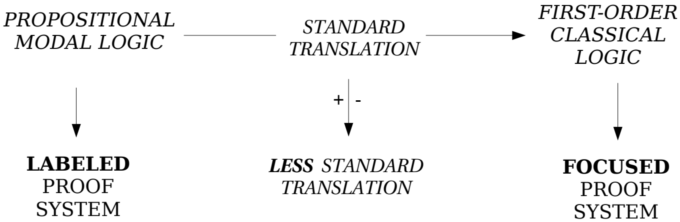
this is a **BIPOLE**

One step back

One step back



One step back (Miller-Volpe, LPAR2015)



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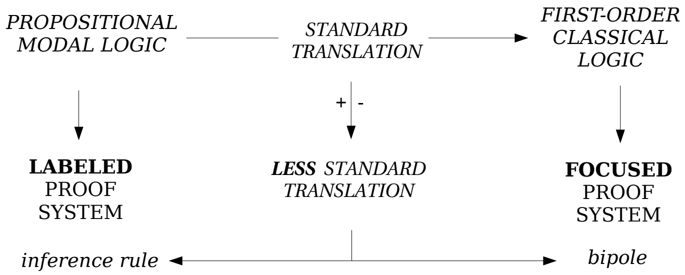

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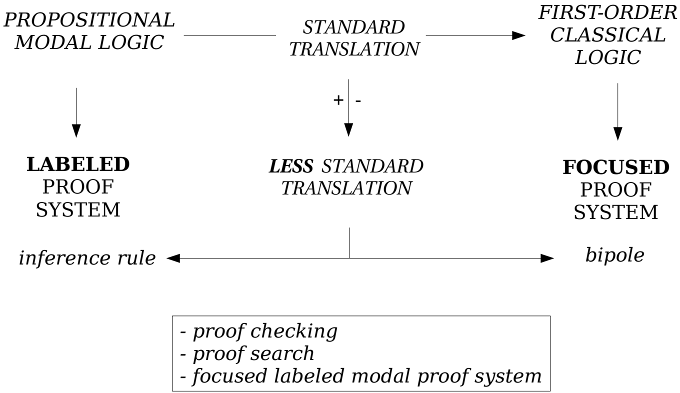


Negative introduction rules

$$\frac{}{\mathcal{G} \vdash \Theta \Uparrow x : t^-, \Gamma}\ t^- \qquad \frac{\mathcal{G} \vdash \Theta \Uparrow \Gamma}{\mathcal{G} \vdash \Theta \Uparrow x : f^-, \Gamma}\ f^- \qquad \frac{\mathcal{G} \vdash \Theta \Uparrow x : A, \Gamma \quad \mathcal{G} \vdash \Theta \Uparrow x : B, \Gamma}{\mathcal{G} \vdash \Theta \Uparrow x : A \wedge^- B, \Gamma}\ \wedge^-$$

$$\frac{\mathcal{G} \vdash \Theta \Uparrow x : A, x : B, \Gamma}{\mathcal{G} \vdash \Theta \Uparrow x : A \vee^- B, \Gamma}\ \vee^- \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash \Theta \Uparrow y : B, \Gamma}{\mathcal{G} \vdash \Theta \Uparrow x : \Box B, \Gamma}\ \Box$$

Positive introduction rules

$$\frac{}{\mathcal{G} \vdash \Theta \Downarrow x : t^+}\ t^+ \qquad \frac{\mathcal{G} \vdash \Theta \Downarrow x : B_1 \quad \mathcal{G} \vdash \Theta \Downarrow x : B_2}{\mathcal{G} \vdash \Theta \Downarrow x : B_1 \wedge^+ B_2}\ \wedge^+$$

$$\frac{\mathcal{G} \vdash \Theta \Downarrow x : B_i}{\mathcal{G} \vdash \Theta \Downarrow x : B_1 \vee^+ B_2}\ \vee^+, i \in \{1, 2\} \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow y : B}{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow x : \Diamond B}\ \Diamond$$

Identity rules

$$\frac{}{\mathcal{G} \vdash x : \neg P_a, \Theta \Downarrow x : P_a}\ \mathit{init} \qquad \frac{\mathcal{G} \vdash \Theta \Uparrow x : B \quad \mathcal{G} \vdash \Theta \Uparrow x : \neg B}{\mathcal{G} \vdash \Theta \Uparrow \cdot}\ \mathit{cut}$$

Structural rules

$$\frac{\mathcal{G} \vdash \Theta, x : C \Uparrow \Gamma}{\mathcal{G} \vdash \Theta \Uparrow x : C, \Gamma}\ \mathit{store} \qquad \frac{\mathcal{G} \vdash \Theta \Uparrow x : N}{\mathcal{G} \vdash \Theta \Downarrow x : N}\ \mathit{release} \qquad \frac{\mathcal{G} \vdash x : P, \Theta \Downarrow x : P}{\mathcal{G} \vdash x : P, \Theta \Uparrow \cdot}\ \mathit{decide}$$

Motivations

Provide (by combining already known formalism interrelation results and some ideas from focusing) a general framework for:

- comparing formalisms;
- proof checking;
- generating new modal proof systems.

One step forward (this paper and more...)

MODAL PROOF SYSTEMS

One step forward (this paper and more...)

MODAL PROOF SYSTEMS

ORDINARY SEQUENTS

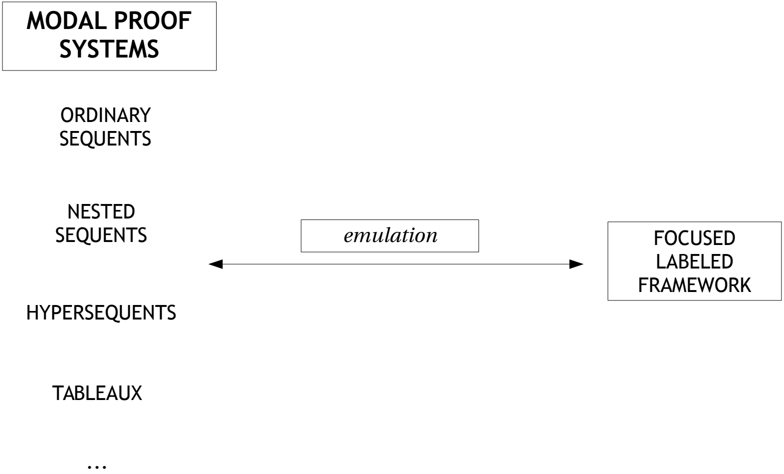
NESTED SEQUENTS

HYPERSEQUENTS

TABLEAUX

...

One step forward (this paper and more...)



One step forward (this paper and more...)

MODAL PROOF SYSTEMS

ORDINARY SEQUENTS

NESTED SEQUENTS

HYPERSEQUENTS

TABLEAUX

...

emulation ⟷

FOCUSED LABELED FRAMEWORK

+ superpowers

One step forward (this paper and more...)

MODAL PROOF SYSTEMS

ORDINARY SEQUENTS

NESTED SEQUENTS

HYPERSEQUENTS

TABLEAUX

...

emulation

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proper polarization

superpower parameters

FOCUSED LABELED FRAMEWORK

+ superpowers

An ordinary sequent system for modal logic

Identity and structural rules

$$\frac{}{\vdash \Gamma, P, \neg P}\ init \qquad \frac{\vdash \Gamma, A \quad \vdash \Delta, \neg A}{\vdash \Gamma, \Delta}\ cut \qquad \frac{\vdash \Gamma, A, A}{\vdash \Gamma, A}\ contr$$

Classical connectives rules

$$\frac{\vdash \Gamma, A \quad \vdash \Gamma, B}{\vdash \Gamma, A \wedge B}\ \wedge \qquad \frac{\vdash \Gamma, A, B}{\vdash \Gamma, A \vee B}\ \vee \qquad \frac{\vdash \Gamma}{\vdash \bot, \Gamma}\ \bot \qquad \frac{}{\vdash \top, \Gamma}\ \top$$

Modal rules

$$\frac{\vdash \Gamma, A}{\vdash \Diamond\Gamma, \Box A, \Delta}\ \Box_K$$

What happens with ordinary sequent systems?

The case of K

$$\frac{\vdash \Gamma, A}{\vdash \Diamond\Gamma, \Box A, \Delta}\ \Box_K$$

This rule works at the same time on $\Box$s and $\Diamond$s.

What happens with ordinary sequent systems?

The case of K

$$\frac{\vdash \Gamma, A}{\vdash \Diamond\Gamma, \Box A, \Delta}\ \Box_K$$

This rule works at the same time on $\Box$s and $\Diamond$s.

~~**Bipole!**~~

What happens with ordinary sequent systems?

- Correspondence between ordinary and labeled sequents:
  - ordinary **classical rules** operate on a single world;
  - ordinary **modal rules** move from one world to another.

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i) Classical reasoning in a world x.

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ii) Modal rule moving from x to y.

i) Classical reasoning in a world x.

What happens with ordinary sequent systems?

- Correspondence between ordinary and labeled sequents:
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  - ordinary **modal rules** move from one world to another.

iii) Classical reasoning in y.

ii) Modal rule moving from x to y.

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$$\frac{\vdash \Gamma, \mathbf{A}}{\vdash \Diamond\Gamma, \Box\mathbf{A}, \Delta}\ \Box_K$$

$$\frac{\mathcal{G} \cup \{\mathbf{xRy}\} \vdash \Sigma, x : \Diamond\Gamma \Uparrow \mathbf{y} : \mathbf{A}}{\mathcal{G} \vdash \Sigma, x : \Diamond\Gamma \Uparrow \mathbf{x} : \Box\mathbf{A}}$$

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One **bipole** for the $\Box$-formula.

What happens with ordinary sequent systems?

The rule for K

$$\frac{\vdash \Gamma, A}{\vdash \Diamond\Gamma, \Box A, \Delta}\ R\Box$$

$$\begin{array}{c} \mathcal{G} \cup \{xRy\} \vdash \Sigma, x : \Diamond\Gamma, y : A \Uparrow \mathbf{y} : \Gamma \\ \vdots \\ \mathcal{G} \cup \{xRy\} \vdash \Sigma, x : \Diamond\Gamma, y : A \Downarrow \mathbf{x} : \Diamond\Gamma \end{array}$$

What happens with ordinary sequent systems?

The rule for K

$$\frac{\vdash \Gamma, A}{\vdash \Diamond\Gamma, \Box A, \Delta}\ R\Box$$

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Multifocusing: the $\Diamond$s can be processed in parallel.

One **bipole** for the $\Diamond$-formulas.

Which superpowers do we need?

Requirements

Superpowers

$$\frac{\mathcal{G} \vdash \Theta \Downarrow x : A}{\mathcal{G} \vdash \Theta \Uparrow \cdot}\ decide^{*} \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow y : B}{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow x : \Diamond B}\ \Diamond$$

* $x : A \in \Theta$

Which superpowers do we need?

Requirements

1. More $\Diamond$s at the same time.

Superpowers

$$\frac{\mathcal{G} \vdash \Theta \Downarrow x : A}{\mathcal{G} \vdash \Theta \Uparrow \cdot} \; decide^{*} \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow y : B}{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow x : \Diamond B} \; \Diamond$$

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$^{*}\ \Omega \subseteq \Theta$

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Requirements

1. More $\Diamond$s at the same time.
2. All formulas associated to such $\Diamond$s move to the same world.

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Superpowers

1. Multifocusing.
2. Attach a "future" to formulas.

$$\frac{\mathcal{G} \vdash \Theta \Downarrow \Omega}{\mathcal{G} \vdash \Theta \Uparrow \cdot}\ decide^{*} \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow y : B, \Omega}{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow x : \Diamond B, \Omega}\ \Diamond$$

* $\Omega \subseteq \Theta$

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Superpowers

1. Multifocusing.
2. Attach a "future" (sequence σ of labels) to formulas.

$$\frac{\mathcal{G} \vdash \Theta \Downarrow \Omega}{\mathcal{G} \vdash \Theta \Uparrow \cdot} \; decide^{*} \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow y\sigma : B, \Omega}{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow xy\sigma : \Diamond B, \Omega} \; \Diamond$$

* if $x\sigma : A \in \Omega$ then $x : A \in \Theta$

Which superpowers do we need?

Requirements

1. More $\Diamond$s at the same time.
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3. Once we move to a new world, we forget about the old ones.

Superpowers

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$$\frac{\mathcal{G} \vdash \Theta \Downarrow \Omega}{\mathcal{G} \vdash \Theta \Uparrow \cdot}\ \textit{decide}^* \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow y\sigma : B, \Omega}{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow xy\sigma : \Diamond B, \Omega}\ \Diamond$$

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Superpowers

1. Multifocusing.
2. Attach a "future" (sequence σ of labels) to formulas.
3. Decorate each sequent with a "present" (set of "active" worlds).

$$\frac{\mathcal{G} \vdash \Theta \Downarrow \Omega}{\mathcal{G} \vdash \Theta \Uparrow \cdot}\ \textit{decide}^{*} \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow y\sigma : B, \Omega}{\mathcal{G} \cup \{xRy\} \vdash \Theta \Downarrow xy\sigma : \Diamond B, \Omega}\ \Diamond$$

* if $x\sigma : A \in \Omega$ then $x : A \in \Theta$

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Superpowers

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$$\frac{\mathcal{G} \vdash_{\mathcal{H}'} \Theta \Downarrow \Omega}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow \cdot}\ \textit{decide}^* \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash_{\mathcal{H}} \Theta \Downarrow y\sigma : B, \Omega}{\mathcal{G} \cup \{xRy\} \vdash_{\mathcal{H}} \Theta \Downarrow xy\sigma : \Diamond B, \Omega}\ \Diamond$$

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Asynchronous introduction rules

$$\frac{}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : t^-, \Omega}\ t^- \qquad \frac{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow \Omega}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : f^-, \Omega}\ f^-$$

$$\frac{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : A, \Omega \quad \mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : B, \Omega}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : A \wedge^- B, \Omega}\ \wedge^- \qquad \frac{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : A, x : B, \Omega}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : A \vee^- B, \Omega}\ \vee^-$$

$$\frac{\mathcal{G} \cup \{xRy\} \vdash_{\mathcal{H}} \Theta \Uparrow y : B, \Omega}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : \Box B, \Omega}\ \Box$$

Synchronous introduction rules

$$\frac{}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Downarrow x\sigma : t^+}\ t^+ \qquad \frac{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Downarrow x\sigma : B_1, \Omega_1 \quad \mathcal{G} \vdash_{\mathcal{H}} \Theta \Downarrow x\sigma : B_2, \Omega_2}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Downarrow x\sigma : B_1 \wedge^+ B_2, \Omega_1, \Omega_2}\ \wedge^+$$

$$\frac{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Downarrow x\sigma : B_i, \Omega}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Downarrow x\sigma : B_1 \vee^+ B_2, \Omega}\ \vee^+, i \in \{1,2\} \qquad \frac{\mathcal{G} \cup \{xRy\} \vdash_{\mathcal{H}} \Theta \Downarrow y\sigma : B, \Omega}{\mathcal{G} \cup \{xRy\} \vdash_{\mathcal{H}} \Theta \Downarrow xy\sigma : \Diamond B, \Omega}\ \Diamond$$

Identity rules

$$\frac{}{\mathcal{G} \vdash_{\mathcal{H}} x : \neg P_a, \Theta \Downarrow x : P_a}\ init \qquad \frac{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : B \quad \mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : \neg B}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow \cdot}\ cut$$

Structural rules

$$\frac{\mathcal{G} \vdash_{\mathcal{H}} \Theta, x : C \Uparrow \Omega}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow x : C, \Omega}\ store \qquad \frac{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow \Omega'}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Downarrow \Omega}\ release \qquad \frac{\mathcal{G} \vdash_{\mathcal{H}'} \Theta \Downarrow \Omega}{\mathcal{G} \vdash_{\mathcal{H}} \Theta \Uparrow \cdot}\ decide$$

The general framework LMF_*^X

Parameters of the framework

- X is a subset of relational properties in $\{T, 4, 5, B, D\}$.
- $*$ can be instantiated in a specific way by the following parameters (of the decide rule):
  1. restrictions on the class of formulas on which multifocusing can be applied;
  2. restrictions on the definition of the future σ of formulas in Ω;
  3. restriction of the present $\mathcal{H}'$.

Theorem

The framework LMF_*^X is sound and complete with respect to the logic KX, for any polarization of formulas.

By playing with polarization and parameters, one can obtain different systems.

Emulation of ordinary sequent systems

$$
\begin{array}{rcl}
[P] & = & P \\
[\neg P] & = & \neg P \\
[A \wedge B] & = & \partial^{+}([A]) \wedge^{-} \partial^{+}([B]) \\
[A \vee B] & = & \partial^{+}([A]) \vee^{-} \partial^{+}([B]) \\
[\Box A] & = & \Box(\partial^{+}([A])) \\
[\Diamond A] & = & \Diamond(\partial^{-}(\partial^{+}([A])))
\end{array}
$$

$$\frac{\mathcal{G} \vdash_{\{y\}} \Theta \Downarrow \Omega}{\mathcal{G} \vdash_{\{x\}} \Theta \Uparrow \cdot} \; decide_{OS}$$

where we have that either:

1. there exists y s.t.:
   - $xRy \in \mathcal{G}$;
   - formulas in Ω have the form $xy : \Diamond A$; or
2. $\Omega = \{x : A\}$ for some A and $x = y$.

Emulation of ordinary sequent systems

Theorem

Derivations in ordinary sequents are emulated by LMF_{OS}^{X}, according to a proper interpretation of sequents.
E.g., for K, a modal rule corresponds to two bipoles.

Corollary

The restriction of the system is complete.

A look at nested sequents

- Same polarization as for ordinary sequents.
- No need for multifocusing.
- No need for restrictions on futures.
- The present is always the set of all labels.

$$\frac{\mathcal{G} \vdash_{\mathcal{L}} \Theta \Downarrow x : A}{\mathcal{G} \vdash_{\mathcal{L}} \Theta \Uparrow \cdot} \; decide_{NS}$$

Conclusion

- We showed the case of K; but it works for geometric extensions.
- Emulation of modal focused systems (e.g., [Lellmann-Pimentel, 2015] or [Chaudhuri-Marin-Strassburger, 2016]).
- What about hypersequents?
  - the present is a multiset;
  - external structural rules as operations on such a present;
  - modal communication rules as a combination of relational and modal rules.
- Not necessarily for emulation: design of new focused calculi.
- Superpowers can be implemented in the augmented version of the focused system LKF used in the project ProofCert.

Thank you!