Alexander Chagrov

1957–2016

1987. PhD Thesis

Complexity of Approximability for Modal and Superintuitionistic Logics

Supervisor: Max Kanovich

1997. Book in Oxford University Press

Alexander Chagrov & Michael Zakharyaschev. Modal Logic

1998. Habilitation Thesis

Modeling of computational processes by means of propositional logic

PhD Students: Mikhail Rybakov (2005), Igor Gorbunov (2006)

Decidability of properties of modal logics

Problem: given a modal logic L, determine whether

- L is decidable
- L is Kripke complete
- L has the finite model property
- L has interpolation
- ...

Kuznetsov (unpublished): not-trivial properties of **recursively axiomatisable** modal logics are undecidable
(see Chagrov 1992)

Thomason (1982): Kripke completeness of **finitely axiomatisable** logics in NExt**K** is undecidable

But
- consistency is decidable in NExt**K** (Makinson 1971)
- interpolation is decidable in NExt**S4** (Maksimova 1979)
- tabularity in NExt**S4**, NExt**GL** (Maksimova, Esakia & Meskhi, Blok)

Chagrov & Chagrova (1990s): a general method for showing undecidability of

- decidability, completeness, FMP, etc. in Ext**Int**, NExt**S4**
- interpolation in NExt**GL**
- first-order definability over **S4** (2006: a simpler proof over **K**)
- ...

Decidability of properties of logics: known results

property	Ext**Int**	NExt**S4**	NExt**GL**	NExt**S4**	NExt**K4**
consistency	✓	✓	✓	✓	✓
decidability	✗	✗	✗	✗	✗
hereditary decidability	?	?	?	?	?
finite model property	✗	✗	✗	✗	✗
Kripke completeness	✗	✗	✗	✗	✗
Post completeness	✓	✓	✓	✓	✓
Hallden completeness	✗	✗	✓	✗	✗
structural completeness	?	?	?	?	?
interpolation	✓	✓	✗	✗	✗
tabularity	✓	✓	✓	✓	?
pre-tabularity	✓	✓	✓	✓	?
local tabularity	?	✓	✓	✓	?
...					

Independent axiomatisability

Old problem: does every normal modal or intermediate logic have an **independent** set of axioms?

(Citkin's problem in Logic Notebook, 1986)

Blok's problem (1980): are the lattices NExt**S4** and Ext**Int** strongly coatomic?

every proper interval $[L_2, L_1]$ contains an immediate predecessor of L_1

Chagrov's key observation: if L_1 has an independent axiomatisation then, for every finitely axiomatisable $L_2 \subset L_1$, there is an immediate predecessor of L_1 in $[L_2, L_1]$

Both Ext**Int** and NExt**S4** contain logics without independent axiomatisations

these lattice are not strongly coatomic

(Chagrov & Zakharyaschev 1995) see open problems

Complexity problems (Chagrov's PhD)

Complexity function for a logic L: $$f_L(n) = \max_{\substack{|\varphi|<n \\ \varphi\notin L}} \min_{\substack{\mathfrak{F}\models L \\ \mathfrak{F}\not\models\varphi}} |\mathfrak{F}|$$

Kuznetsov 1975: is $f_{\mathrm{Int}} = poly(n)$? (or does **Int** have the polynomial FMP?)
in which case **Int** and **Cl** would be polynomially equivalent and NP = PSPACE

Chagrov 1985:

linear FMP: all logics containing **S4.3**

polynomial FMP: minimal logics of finite width in NExt**K4** and Ext**Int**

minimal logics of finite depth NExt**K4** and Ext**Int**

(all these logics are polynomially equivalent to **Cl**)

fast growing FMP: for any function $f(n)$, there are logics L of width 1 in NExt**K4** and of width 2 in Ext**Int** with FMP and such that $f_L(n) \geq f(n)$

Chagrov & M. Rybakov 2003: **K4**(0), **Grz**(1), **GL**(1) do not have polynomial FMP and are PSPACE- complete

Tabularity

Chagrov's tabularity criterion: a logic L in Ext**K** is tabular iff $tab_n \in L$, for some $n < \omega$

$$tab_n = \alpha_n \wedge alt_n$$

$$\alpha_n = \neg(\varphi_1 \wedge \Diamond(\varphi_2 \wedge \Diamond(\varphi_3 \wedge \ldots \wedge \Diamond\varphi_n) \ldots))$$

$$\varphi_i = p_1 \wedge \ldots \wedge p_{i-1} \wedge \neg p_i \wedge p_{i+1} \wedge \ldots \wedge p_n$$

(a similar criterion for multi-modal logic).

The semantic condition for α_n is:

$\neg\exists x_1, \ldots, x_n (x_1 R \ldots R x_n$ & all x_i are different).

Remember, however, that **tabularity** in (N)Ext**K** is **undecidable** (**Chagrov 1996**).

Local tabularity in NExt**K** is also **undecidable** (**Chagrov 2002**).

Chagrov's conjecture (1994): $\mathrm{K} + \alpha_n$ is locally tabular.

Shehtman proved this conjecture in 2014.

Post completeness

A logic L is **Post complete** in Ext**K** is L if consistent and
has no proper consistent extensions in Ext**K**

Chagrov 1985: L is **generally Post complete** if L is consistent and has no
consistent extensions closed under the rules admissible in L

> For every generally Post complete modal logic L,
> L is Post complete in Ext**K** iff L is structurally complete

- there is a continuum of generally Post complete logics in NExt**K4**
- there is a continuum of Post complete logics in Ext**K4**

A logic is **antitabular** if it is consistent but does not have finite models
(a consistent logic is antitabular iff all its Post complete extensions are not tabular)

> If $L \supseteq \mathrm{K4}$ has infinitely many Post complete extensions then
> it also has an antitabular extension

Other results

Chagrov 1992: There exists a continuum of maximal intermediate propositional logics with the disjunction property.

(Maksimova 1984: there exist infinitely many such logics. The only known explicit example is Medvedev logic.)

Chagrov 2015: There exists a normal modal logic $\boldsymbol{L}$ with the FMP and a variable-free formula φ such that $\boldsymbol{L} + \varphi$ lacks the FMP.