Before announcement

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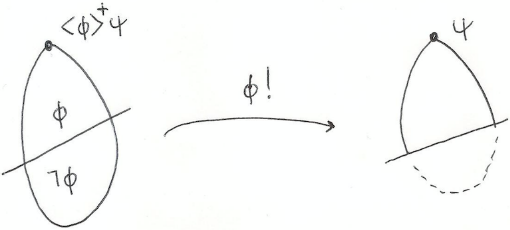
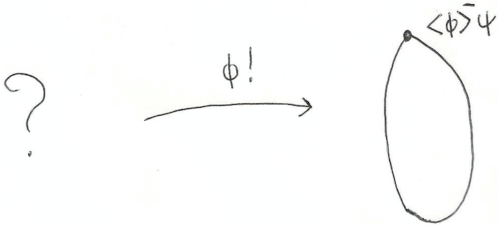
Outline

Introduction

Public Announcement Logic (*PAL*)



Other direction



Introduction

Public Announcement Logic (*PAL*)

- $[\varphi]^+\psi$: "**After** φ's announcement, ψ is true"
- $\langle\varphi\rangle^+\psi ::= \neg[\varphi]^+\neg\psi$

Other direction

- $[\varphi]^-\psi$: "**Before** φ's announcement, ψ was true"
- $\langle\varphi\rangle^-\psi ::= \neg[\varphi]^-\neg\psi$

Remarks

- $\models \psi \to [\varphi]^+\langle\varphi\rangle^-\psi$
- $\models \psi \to [\varphi]^-\langle\varphi\rangle^+\psi$
- $\models \langle\varphi\rangle^+\psi \to [\varphi]^+\psi$
- $\not\models \langle\varphi\rangle^-\psi \to [\varphi]^-\psi$

Introduction

History-based structures

- G. Aucher, A. Herzig (2007)
- B. Renne, J. Sack, A. Yap (2009)
- J. Sack (2008)

Storing the values of formulas

- H. van Ditmarsch, J. van Eijck, W. Wu (2010)
- H. van Ditmarsch, J. Ruan, R. Verbrugge (2007)

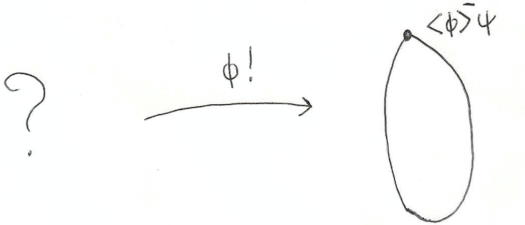
Subset Space Logic

- A. Dabrowski, L. Moss, L., R. Parikh (1996)
- B. Heinemann (2007)

Introduction

Appealing semantics for converse announcement of φ

- Truth in all models of which the current model is the φ restriction



Remark

- $\models \Box p \rightarrow \langle p \rangle^{-} \neg \Box p$

What we chose

- A setting similar to that of Subset Space Logic

Logic with converse announcements

Syntax

- $\varphi, \psi ::= p \mid \bot \mid \neg\varphi \mid (\varphi \vee \psi) \mid \Box\varphi \mid [\varphi]^{+}\psi \mid [\varphi]^{-}\psi$
- $\Diamond\varphi ::= \neg\Box\neg\varphi$
- $\langle\varphi\rangle^{+}\psi ::= \neg[\varphi]^{+}\neg\psi$
- $\langle\varphi\rangle^{-}\psi ::= \neg[\varphi]^{-}\neg\psi$

Readings

- $\Box\varphi$: “The agent **knows** that φ”
- $[\varphi]^{+}\psi$: “**After** φ’s announcement, ψ is true”
- $[\varphi]^{-}\psi$: “**Before** φ’s announcement, ψ was true”

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Models

Structure of the form $\mathcal{M} = (W, X, V)$

- W is a nonempty set (**worlds** x**,** y**, etc**)
- X is a nonempty set of nonempty subsets of W (**steps** S**,** T**, etc**)
- $V : p \mapsto V(p) \subseteq W$ (**valuation**)

Tip

World-step pair (x, S) such that $x \in S$

- x is the real world
- S is the current restriction of the model

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Truth conditions

In model $\mathcal{M} = (W, X, V)$ with respect to tip (x, S)

- $\mathcal{M}, (x, S) \models p$ iff $x \in V(p)$
- $\mathcal{M}, (x, S) \not\models \bot$
- $\mathcal{M}, (x, S) \models \neg\varphi$ iff $\mathcal{M}, (x, S) \not\models \varphi$
- $\mathcal{M}, (x, S) \models \varphi \vee \psi$ iff $\mathcal{M}, (x, S) \models \varphi$ or $\mathcal{M}, (x, S) \models \psi$
- $\mathcal{M}, (x, S) \models \Box\varphi$ iff for all $y \in S$, $\mathcal{M}, (y, S) \models \varphi$
- $\mathcal{M}, (x, S) \models [\varphi]^{+}\psi$ iff for all $T \in X$, if $x \in T$ and $T = \{z \in S : \ \mathcal{M}, (z, S) \models \varphi\}$ then $\mathcal{M}, (x, T) \models \psi$
- $\mathcal{M}, (x, S) \models [\varphi]^{-}\psi$ iff for all $T \in X$, if $x \in T$ and $S = \{z \in T : \ \mathcal{M}, (z, T) \models \varphi\}$ then $\mathcal{M}, (x, T) \models \psi$

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- $\mathcal{M}, (x, S) \models [\varphi]^-\psi$ iff for all $T \in X$, if $x \in T$ and $S = \{z \in T : \mathcal{M}, (z, T) \models \varphi\}$ then $\mathcal{M}, (x, T) \models \psi$

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Logic with converse announcements

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Free models

Model which globally satisfies

- $\varphi \to \langle \varphi \rangle^{+} \top$

Remarks

- $\models [\varphi]^{+} p \leftrightarrow (\varphi \to p)$
- $\models [\varphi]^{+} \bot \leftrightarrow \neg \varphi$
- $\models [\varphi]^{+} \neg \psi \leftrightarrow (\varphi \to \neg [\varphi]^{+} \psi)$
- $\models [\varphi]^{+} (\psi \vee \chi) \leftrightarrow [\varphi]^{+} \psi \vee [\varphi]^{+} \chi$
- $\models [\varphi]^{+} \Box \psi \leftrightarrow (\varphi \to \Box [\varphi]^{+} \psi)$

If φ is a $[\cdot]^{-}$-free formula then

- $\models \varphi$ iff $\varphi \in PAL$

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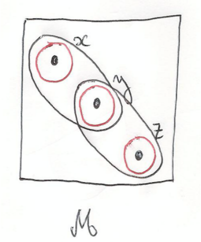
Expressivity

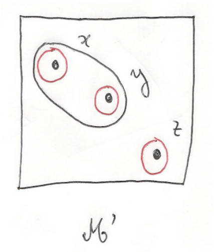
Proposition: There is no $[\cdot]^+$-free formula $\varphi(p,q)$ such that

- $\models \varphi(p,q) \leftrightarrow \langle p\rangle^+\langle q\rangle^-\Diamond(p \wedge \neg q)$

Proof:



$V(p) = \{y,z\}$
$V(q) = \{y\}$



$V'(p) = \{y,z\}$
$V'(q) = \{y\}$

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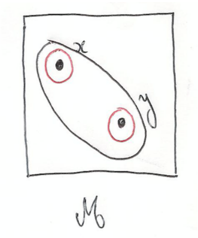
Expressivity

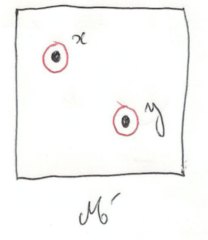
Proposition: There is no $[\cdot]^{-}$-free formula $\varphi(p,q)$ such that

- $\models \varphi(p,q) \leftrightarrow \langle p\rangle^{-}\Diamond q$

Proof:



$V(p) = \{x\}$
$V(q) = \{y\}$



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Axiomatization

(A_1)	all instances of *CPL*
(A_2)	$\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$
(A_3, A_4, A_5)	$\Box\phi \to \phi \qquad \Diamond\phi \to \Box\Diamond\phi \qquad \Box\phi \to \Box\Box\phi$
(A_6)	$[\phi]^+(\psi \to \chi) \to ([\phi]^+\psi \to [\phi]^+\chi)$
(A_7)	$[\phi]^-(\psi \to \chi) \to ([\phi]^-\psi \to [\phi]^-\chi)$
(A_8, A_9)	$\psi \to [\phi]^+\langle\phi\rangle^-\psi \qquad \psi \to [\phi]^-\langle\phi\rangle^+\psi$
(A_{10})	$\langle\phi\rangle^+\psi \to [\phi]^+\psi$
(A_{11}, A_{12})	$\neg\phi \to [\phi]^+\bot \qquad [\phi]^+\bot \to \neg\phi$
(A_{13})	$[\top]^+\phi \to \phi$
(A_{14}, A_{15})	$p \to [\phi]^+p \qquad \neg p \to [\phi]^+\neg p$
(A_{16})	$\langle\phi\rangle^+\Box\psi \to \Box[\phi]^+\psi$
(A_{17})	$\Box[\phi]^+\psi \to [\phi]^+\Box\psi$

Maximal ignorance variant

Model of maximal ignorance

Structure $\mathcal{M}_0 = (W_0, X_0, V_0)$

- $W_0 = 2^{VAR}$
- $X_0 = 2^{2^{VAR}} \setminus \{\emptyset\}$
- $V_0 : p \mapsto V_0(p) \subseteq W_0$ is such that $x \in V_0(p)$ iff $p \in x$

Remarks

- $\mathcal{M}_0$ is free
- If φ is a $\{[\cdot]^+, [\cdot]^-\}$-free formula then $\mathcal{M}_0 \models \varphi$ iff $\varphi \in S5$
- $\mathcal{M}_0 \models \Box p \to \langle p \rangle^- \neg \Box p$
- For all formulas φ, there exists a $\{[\cdot]^+, [\cdot]^-\}$-free formula ψ such that $\mathcal{M}_0 \models \varphi \leftrightarrow \psi$

Conclusion

Open questions

- Validity in $\mathcal{M}_0$ is decidable: complexity?
- Multi-agent variants?
- Extension with the effort modality of *SSL*? or with the converse effort modality of *SSL*?
- Characterization of the pairs (φ, ψ) such that $\models [\varphi]^+\psi$? or such that $\models [\varphi]^-\psi$?

References

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